Generative Models

DISCRIMINATIVE MODELS

DIRECTLY MODEL

$$P[Y = k \mid X = x]$$

- KNN
- TREE
- LOGISTIC IF BINARY

GIVEN MODEL, COULD ONLY GENERATE NEW DATA GIVEN X.

GENERATIVE MODELS

- MODEL FULL JOINT DISTRIBUTION

$$P[Y = k, X = x]$$

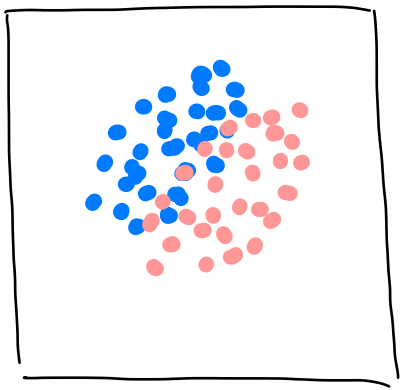
- GIVEN MODEL, COULD GENERATE NEW DATA.

CLASSIFICATION WITH GENERATIVE MODELS

HOW TO MODEL ?

$$P[Y=k \mid X=x] = \frac{P[Y=k,\ X=x]}{P[X=x]}$$

WE KNOW WHAT TO DO FROM HERE.



X_2

X_1

"LOCATION" "SHAPE"

"MEAN" "VARIANCE"

pdf $f_1(x)$

$$(X_1, X_2) \mid Y=1 \sim MVN(\mu_1, \Sigma_1)$$

$$(X_1, X_2) \mid Y=0 \sim MVN(\mu_0, \Sigma_0)$$

pdf $f_2(x)$

$P[Y=1] = \pi_1$ $P[Y=0] = \pi_0$

BAYES THEOREM

$$P[Y=1 \mid X=x] = \frac{\pi_1 f_1(x)}{\pi_0 f_0(x) + \pi_1 f_1(x)}$$

POSTERIOR

π_0, π_1 "PRIOR" PROBABILITIES

$f_1(x), f_2(x)$ LIKELIHOODS

NEED TO ESTIMATE

$$\left.\begin{array}{l} \pi_0, \pi_1 \\ \mu_1, \mu_2 \\ \Sigma_1, \Sigma_2 \end{array}\right\}$$

How? MLE PROBABLY

IN GENRAL, WITH CATEGORIES $g = 1, 2, \ldots, G$

$$p_k(x) = P[Y = k \mid X = x] = \frac{\pi_k f_k(x)}{\sum_{g=1}^{G} \pi_g f_g(x)}$$

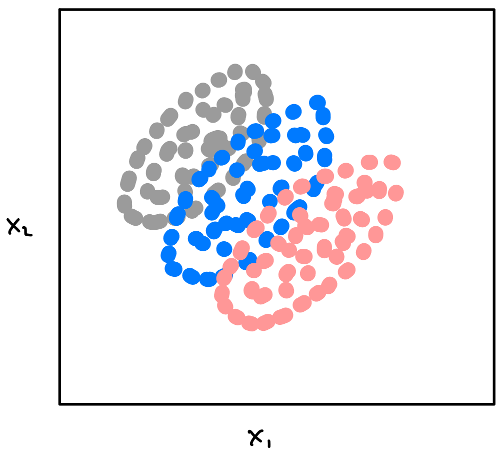
$$X \mid Y = k \sim MVN(\mu_k, \Sigma_k)$$

THREE WAYS TO MODEL $f_k(x)$

LDA

LINEAR

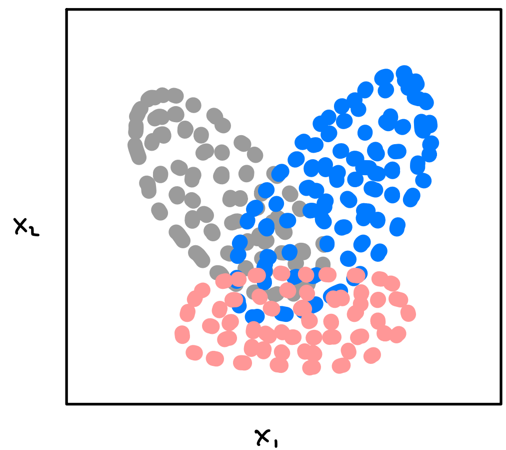
$$\Sigma = \Sigma_1 = \Sigma_2 = \dots = \Sigma_G$$



QDA

QUADRATIC

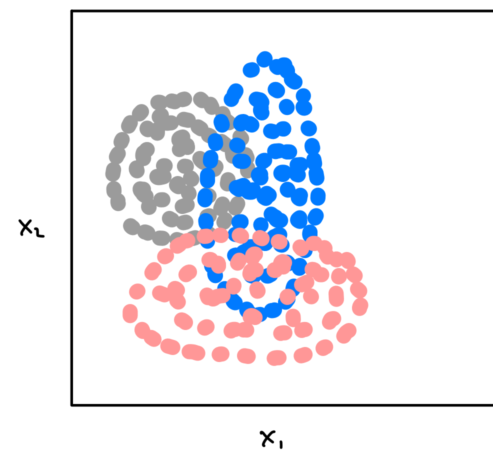
$$\Sigma_k$$



NB

NAIVE BAYES

$$\Sigma_k = \begin{bmatrix} \sigma_{k1}^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_{kp}^2 \end{bmatrix}$$



Naive Bayes

Naive $\Rightarrow$ Given Y, $X_1, \ldots X_p$ ind

Thus

$$f_K(x_1, x_2, \ldots x_p) = \prod_{j=1}^{p} f_{kj}(x_j)$$

pdf of feature j given $Y = k$

need to estimate

$N(\mu_{kj}, \sigma_{kj})$

In R

mass::lda

mass::qda

klaR::Naive Bayes