Generative Models
**Discriminative Models**

Directly model

\[ P[Y = k \mid X = x] \]

- KNN
- TREE
- LOGISTIC IF BINARY

Given model, could only generate new data given \( X \).
Generative Models

- Model full joint distribution

\[ P[y = k, x = x] \]

- Given model, could generate new data.
**Classification with Generative Models**

\[
P[y=k \mid x=x] = \frac{P[y=k, x=x]}{P[x=x]}
\]

*We know what to do from here.*
The diagram illustrates a two-class classification problem with two features, $x_1$ and $x_2$. The classes are represented by blue and red dots.

- $p(y=1) = \pi_1$ for the class $y=1$.
- $p(y=0) = \pi_0$ for the class $y=0$.

The conditional probability density functions (PDFs) are given by

- $f_1(x) | y=1$ follows a multivariate normal distribution $\mathcal{MN}(\mu_1, \Sigma_1)$.
- $f_2(x) | y=0$ follows a multivariate normal distribution $\mathcal{MN}(\mu_0, \Sigma_0)$.

The distribution of the data points is shown with respect to the classes and their means. The shape of the distributions is indicated by ellipses, typical of multivariate normal distributions.
\[ P[Y = 1 | X = x] = \frac{P_1 f_1(x)}{P_0 f_0(x) + P_1 f_1(x)} \]

Bayes' Theorem

\( P_0, P_1 \) "Prior" Probabilities

\( f_1(x), f_2(x) \) Likelihoods

Need to estimate \( P_0, P_1, \mu_1, \mu_2, \Sigma_1, \Sigma_2 \) \}

How? MLE (Maximum Likelihood Estimate)
In general, with categories \( g = 1, 2, \ldots, G \)

\[
P_k(x) = P[Y = k \mid X = x] = \frac{\prod_{g=1}^{G} f_{g}(x)}{\sum_{g=1}^{G} \prod_{j=1}^{g} f_{j}(x)}
\]

\( X \mid Y = k \sim \text{MVN} \left( \mu_k, \Sigma_k \right) \)
Three ways to model $f_k(x)$

- **LINEAR**
  - LDA
  
  $\Sigma = \Sigma_1 = \Sigma_2 \ldots = \Sigma_k$

- **QUADRATIC**
  - QDA
  
  $\Sigma_k$

- **NAIVE BAYES**
  - NB
  
  $\Sigma_k = \begin{bmatrix} \omega_e & 0 \\ 0 & \omega_k \end{bmatrix}$
Naive Bayes

Naive \Rightarrow Given \ Y, \ X_1, \ldots, \ X_p \ \text{ind}

Thus

\[ f_K(x_1, x_2, \ldots, x_p) = \prod_{j=1}^{p} f_{kj}(x_j) \]

\mathcal{N}(\mu_k, \Sigma_k)

\text{pdf of feature } j \text{ given } Y = k

\text{Need to estimate}
In R

mass :: lda

mass :: qda

klq R :: Naive Bayes