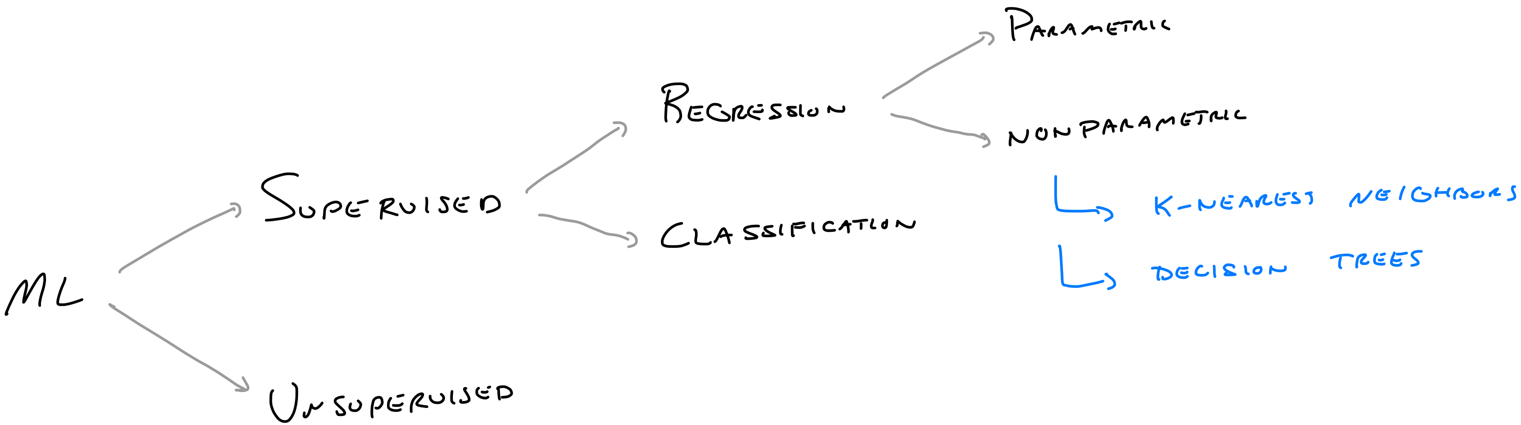
Nonparametric Regression



ML

Supervised

Unsupervised

Regression

Classification

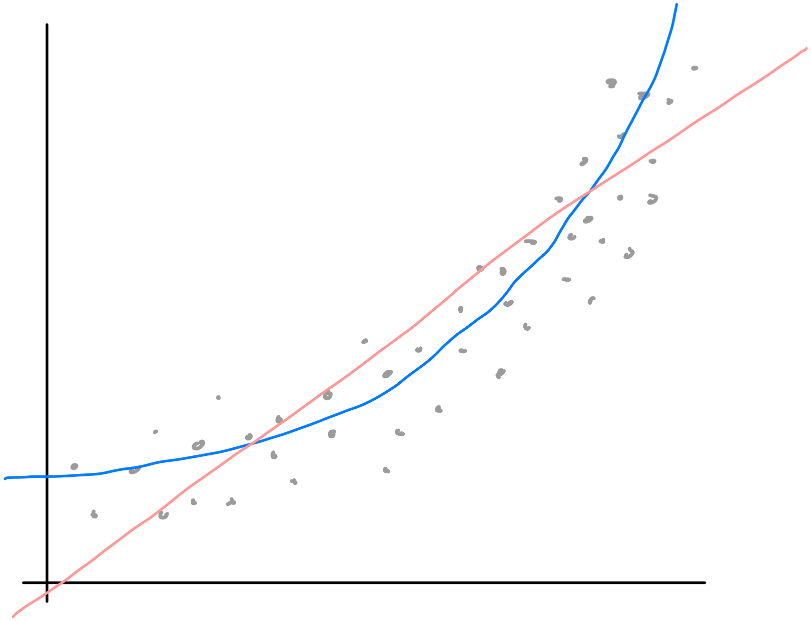
Parametric

Nonparametric

K-Nearest Neighbors

Decision Trees

x	y



Want

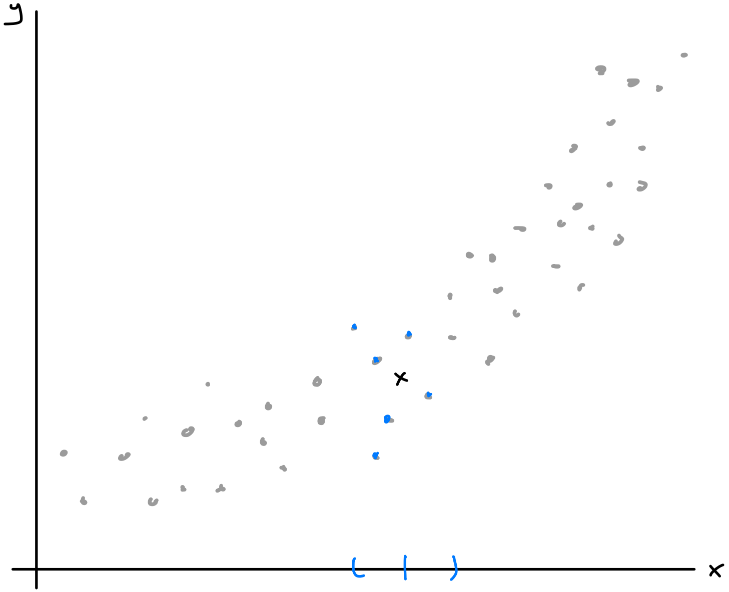
$$\mathbb{E}[Y \mid X = x]$$

Assume

$$Y = \underline{\beta_0 + \beta_1 x_1 + \beta_2 x_2^2} + \varepsilon$$

Assume

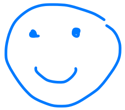
$$Y = \underline{\beta_0 + \beta_1 x_1} + \varepsilon$$



WANT

$$\mathbb{E}[Y \mid X = x]$$

- $\hat{\mathbb{E}}[Y \mid X = x] = \text{AVE}\left(\{y_i \text{ WHERE } x_i = x\}\right)$ ← WON'T WORK
- $\hat{\mathbb{E}}[Y \mid X = x] = \text{AVE}\left(\{y_i \text{ WHERE } x_i \text{ "CLOSE" TO } x\}\right)$

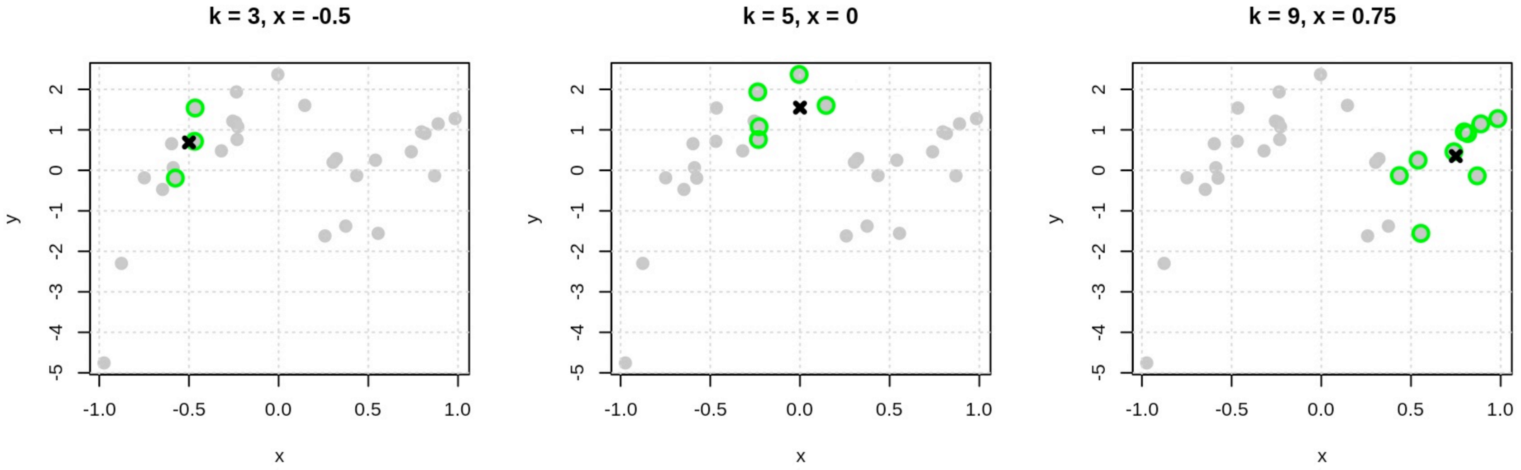


k - Nearest Neighbors

To estimate $\mu(x) = \mathbb{E}[Y \mid X = x]$

use $$\hat{\mu}_k(x) = \frac{1}{k} \underbrace{\sum_{\{i \,:\, x_i \in N_k(x, D)\}}}_{} y_i$$

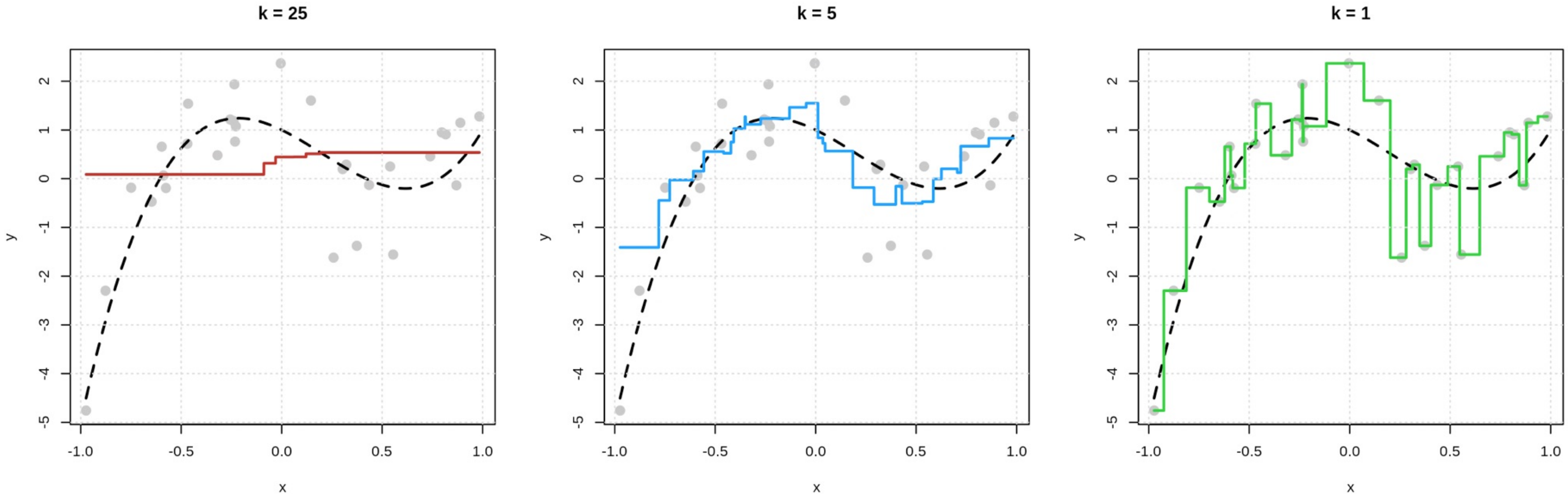
k observations with x_i nearest to x



k = 3, x = -0.5

k = 5, x = 0

k = 9, x = 0.75


k = 25
k = 5
k = 1
y
x
2
1
0
-1
-2
-3
-4
-5
-1.0
-0.5
0.0
0.5
1.0

TUNING PARAMETERS

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$$

MODEL PARAMETERS

→ LEARNED FROM DATA

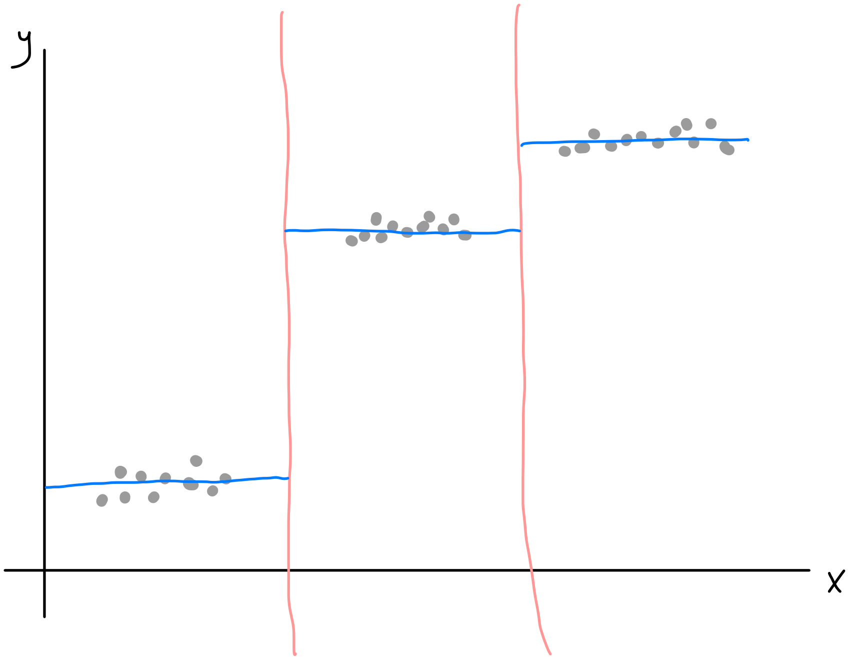
K IN KNN

TUNING PARAMETER

→ DEFINES HOW TO LEARN FROM DATA

OTHER KNN NOTES

- 'FAST' TO TRAIN, "SLOW" TO PREDICT LAZY!
- WHICH FEATURES SHOULD BE USED? ???
- CATEGORICAL FEATURES? DUMMY ENCODING
- HOW TO CALCULATE DISTANCE? YOU PICK!
- FEATURE SCALING? !!!



IDEA: FIND NEIGHBORHOODS, PREDICT AVERAGE OF y_i IN NEIGHBORHOODS

Decision Trees

$$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

Feature + cutoff

Avg y_i in N_L

Avg y_i in N_R

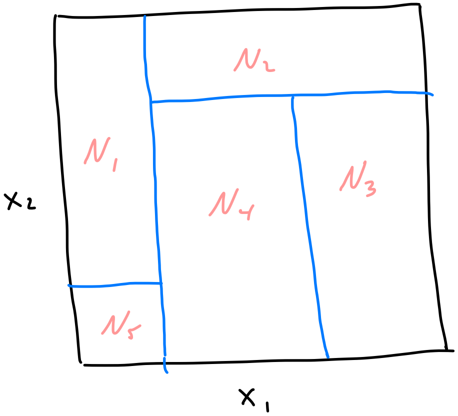
Find "split" that minimizes

$$\sum_{\substack{i \in N_L \\ \downarrow \\ x<c}} (y_i - \hat{\mu}_{N_L})^2 + \sum_{\substack{i \in N_R \\ \downarrow \\ x>c}} (y_i - \hat{\mu}_{N_R})^2$$

Recursive Paritioning

$$\begin{tikzcd}
\sum_{i \in N_L} \left(y_i - \hat{\mu}_{N_L}\right)^2 + & \sum_{i \in N_R} \left(y_i - \hat{\mu}_{N_R}\right)^2 \arrow[dl] \arrow[d] \\
\sum_{i \in N_{R_1}} \left(y_i - \hat{\mu}_{N_{R_1}}\right)^2 + & \sum_{i \in N_{R_2}} \left(y_i - \hat{\mu}_{N_{R_2}}\right)^2
\end{tikzcd}$$

RECURSIVE PARTITIONING



# NEIGHBORHOODS

$$SSE = \sum_{j=1}^{J} \sum_{i \in N_j} \left(y_i - \hat{\mu}_j \right)^2$$

AVE y_i IN N_j

$$R^2 = 1 - \frac{SSE}{SST}$$

How to stop?

rpart::rpart in R

minsplit — ONLY CONSIDER SPLIT IN NEIGHBORHOOD IF IT HAS AT LEAST THIS MANY OBSERVATIONS

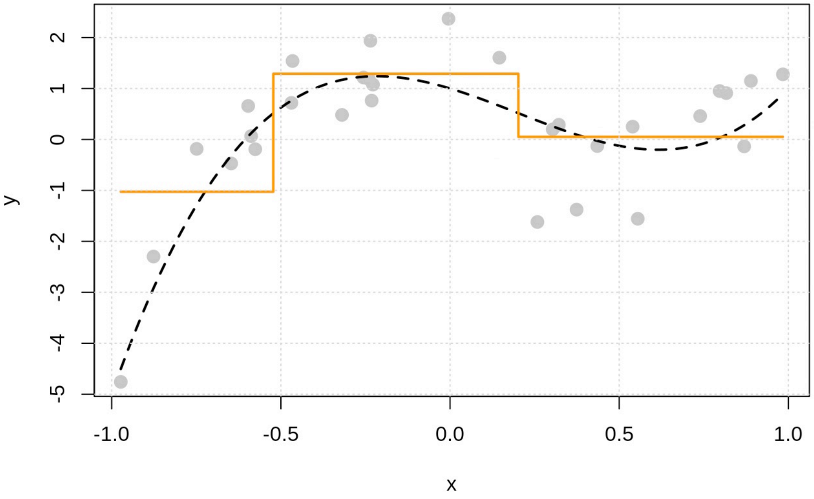
MINSPLIT = 2 ⇒ CAN ALWAYS SPLIT

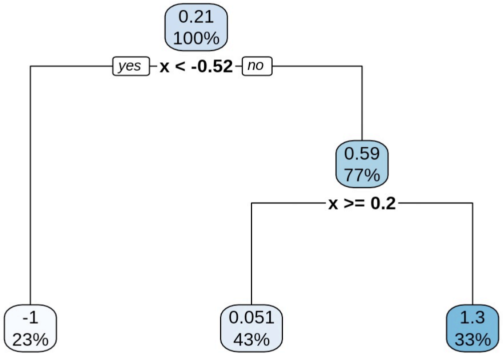
cp — ONLY ACCEPT A SPLIT IF IT INCREASES R^2 BY THIS AMOUNT OR MORE

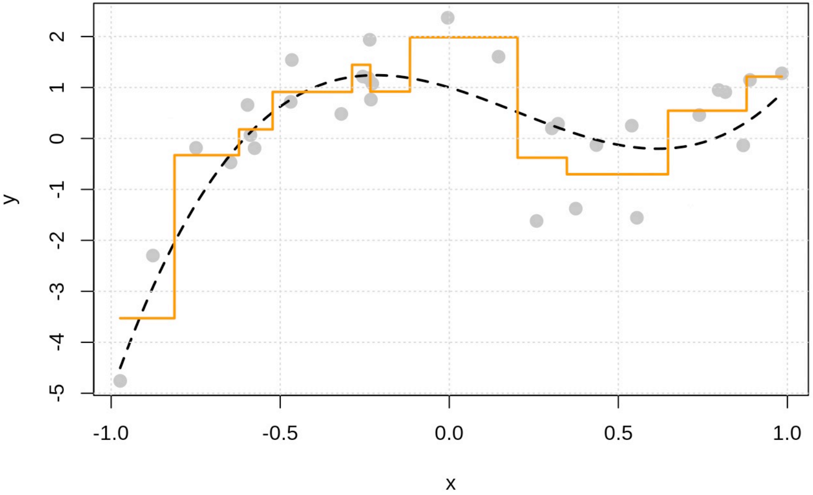
↑

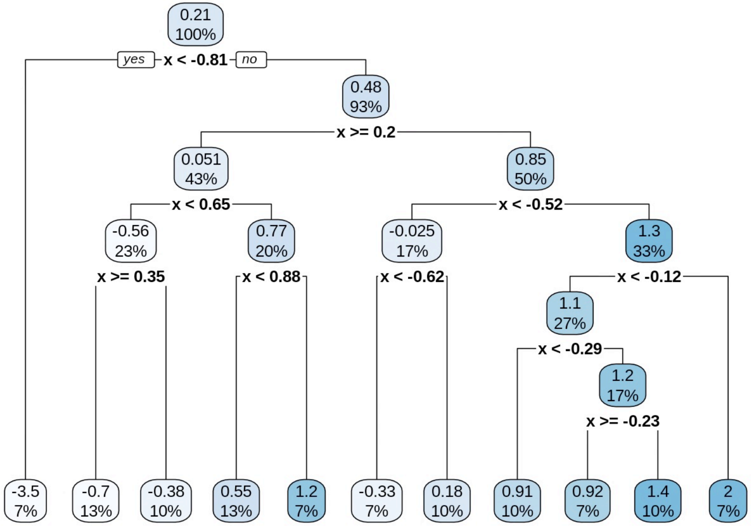
"COMPLEXITY PARAMETER"

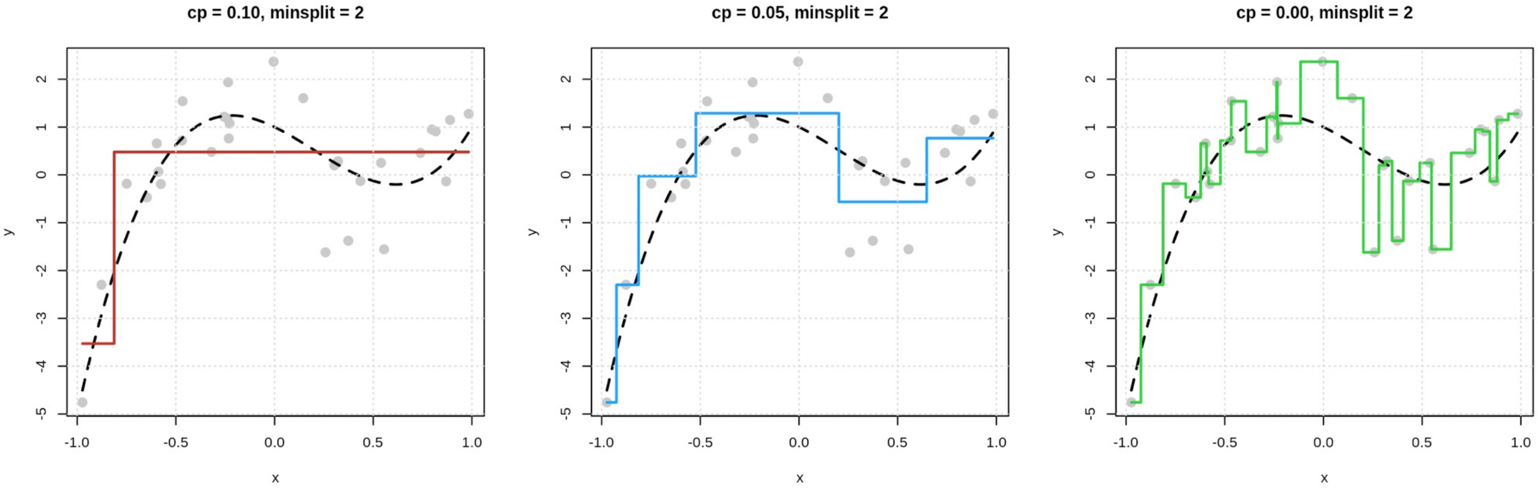
CP = 0 ⇒ ANY SPLIT WILL BE ACCEPTED

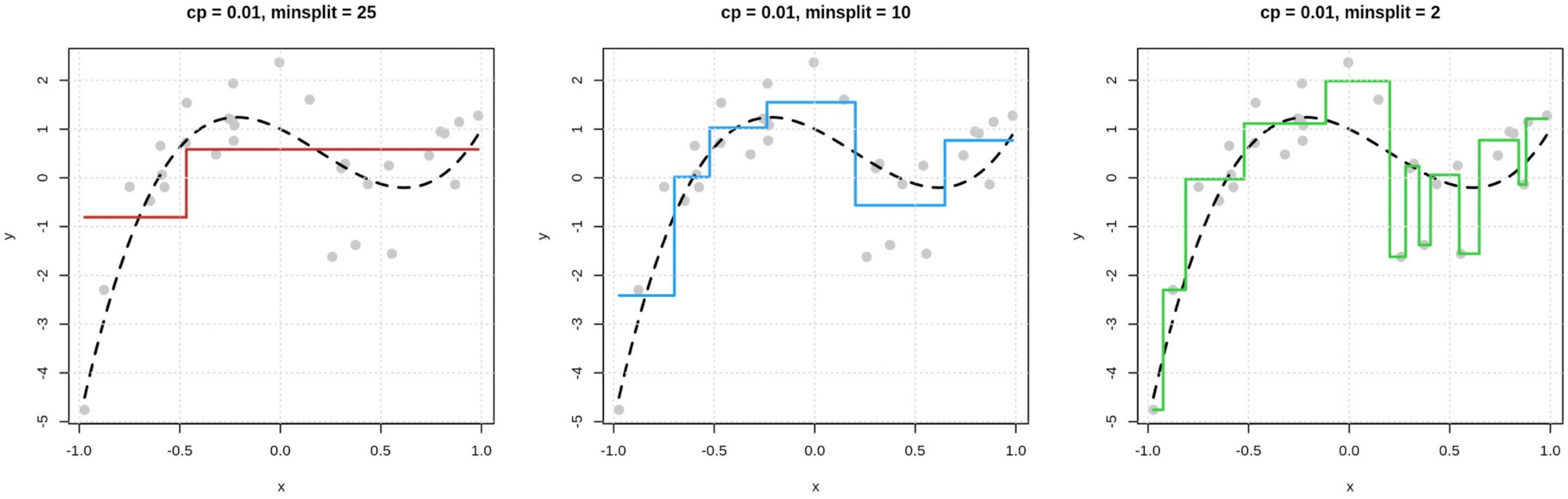

2
1
0
-1
-2
-3
-4
-5
y
-1.0
-0.5
0.0
0.5
1.0
x


0.21
100%
yes
x < -0.52
no
0.59
77%
x >= 0.2
-1
23%
0.051
43%
1.3
33%


y
2
1
0
-1
-2
-3
-4
-5
-1.0
-0.5
0.0
0.5
1.0
x


0.21
100%
yes
x < -0.81
no
0.48
93%
x >= 0.2
0.051
43%
0.85
50%
x < 0.65
x < -0.52
-0.56
23%
0.77
20%
-0.025
17%
1.3
33%
x >= 0.35
x < 0.88
x < -0.62
x < -0.12
1.1
27%
x < -0.29
1.2
17%
x >= -0.23
-3.5
7%
-0.7
13%
-0.38
10%
0.55
13%
1.2
7%
-0.33
7%
0.18
10%
0.91
10%
0.92
7%
1.4
10%
2
7%


cp = 0.10, minsplit = 2
cp = 0.05, minsplit = 2
cp = 0.00, minsplit = 2
y
x
-5
-4
-3
-2
-1
0
1
2
-1.0
-0.5
0.0
0.5
1.0


cp = 0.01, minsplit = 25
cp = 0.01, minsplit = 10
cp = 0.01, minsplit = 2
y
x
2
1
0
-1
-2
-3
-4
-5
-1.0
-0.5
0.0
0.5
1.0